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ABSTRACT

The present note employs some simple tools of the theory of decision-making under uncertainty to provide a possible theoretical basis for the empirically observed phenomenon that the tenurial arrangement which predominates in areas of relatively low productivity is share-cropping, while that which predominates in areas of relatively high productivity is fixed-renting. In a minor extension of the main model presented in this note, an attempt is made to establish how, under competitive conditions, the share-rental rate will settle at one-half - another empirically observed phenomenon in the Indian context. The principal ingredients of analysis are constituted by the assumptions of expected-utility-maximization and constant relative risk-aversion on the part of agents (landlords and tenants).

Production Risks and the Choice of Mode of Tenancy :

A Theoretical Note

by

S. Subramanian*

Introduction

An attempt is made in this note to examine the impact of production-risk on the choice made by landlords and tenants between two modes of tenancy : share-cropping and fixed-renting. Cheung /1/ has furnished some evidence from China to support the possibility that as production risks become more important, there is a tendency for the incidence of share-tenancy to increase. This tendency seems to be a feature also of the State of Tamil Nadu, as the following Table, based on data from the World Agricultural Census (1970-71)[1] suggests:

Table 1 : Area under Different Types of Tenancy (Tamil Nadu, 1970-71)

	(1) Extent of Land Rented from others (hectares)	(2) Extent of Land Rented under share of produce (hectares)	(3) Extent of Land Rented under Fixed produce (hectares)	(4) Ratio of (2) to (1) (percent)	(5) Ratio of (3) to (1) (percen
For Drought-Prone Areas	18250	10570	2250	57.92	12.33
For Non-Drought Prone Areas	217715	55170	102769	25.34	47.20

Table 1 reveals that the incidence of share-cropping is much larger in the drought-prone areas of Tamil Nadu than in the non-drought-prone areas, and conversely that the

* I wish to express my deep gratitude to Professor T.N. Srinivasan and Professor U. Shankar for comments on earlier drafts of this note. The usual *caveat* applies.

incidence of tenancy under fixed produce is much smaller in the drought-prone areas than in the non-drought-prone areas. In seeking a theoretical rationale for the phenomenon of share-cropping yielding place to fixed-renting as production risks diminish, I shall employ certain standard tools of portfolio choice theory to elicit some simple relationships between risk and tenancy. The emphasis will be on very considerable abstraction from complexities, the principal object of the exercise being to identify, within a logical framework, the consequences of positing (expected) utility-maximizing behaviour on the part of agents (landlords and tenants). This shall be my chief concern in Section I of this note. In Section II, a minor extension of the basic framework employed in Section I is undertaken to demonstrate how, under competitive conditions, the equilibrium share-rental rate will settle at one-half -- another empirical phenomenon which has been observed in the Indian context.[2]

II The Choice between share-cropping and fixed-rent tenancy under risk

Certain simplifying aspects of the setting within which I shall work are specified below:

(1) It is assumed, for the purposes of this Section, that the landlord has already resolved the problem of choice between renting out his land and not renting it out in favour of the former, and likewise that the tenant has resolved the problem of choice between renting in land and not renting it in in favour of the former: the choice problem of each agent is effectively confined to that of deciding between the share-cropping and fixed-renting regimes of tenancy.

(2) The share-of-produce accruing to the landlord under share-cropping will be taken as being regarded parametrically by the tenant and the landlord; one is not, in this section, concerned with the mechanism by which the share is determined.

(3) Under fixed-renting, it will be assumed that the landlord is entitled to the fixed-rental or to the entire output raised by the tenant, whichever is smaller. Problems of moral hazard and untruthful revelation are assumed away.

(4) For the purposes of this note, an increase in the productivity of land will be associated with an increase in the expected output per unit size of land and a dimunition in the fluctuation of output from its expected value. To give this operational meaning, one could regard productivity to be an increasing function of the reliability of irrigation -- where increasing reliability is understood to mean both the availability of a larger quantum of water per unit time and a smaller variation in this quantum from its expected value.

(5) Finally, the assumption is made that both the landlord and the tenant carry out their decision-making activity under conditions of identical information-availability and that both interpret the information available to them in identical fashion.

Other simplifying assumptions will be introduced as we go along. With these preliminary remarks, we can turn to some formal aspects of the note.

Imagine a landlord with a piece of land of unit size, which is available for being leased out on either a fixed-rent or share-cropping basis. The rental share due to the landlord under share-cropping is designated by r ($r \in (0,1)$) and that of the tenant is designated by ρ ($\equiv (1-r)$). $\{\theta_i\}$ is an n-tuple of states of nature where each state of nature may be taken to correspond to a particular level of availability of water from irrigation. $X = (X_1, X_2, \ldots, X_n)$ is an n-dimensional vector whose typical element X_i denotes the quantity of output which obtains under state of nature θ_i. It will be assumed, without loss of generality, that $X_n > X_{n-1} > \cdots > X_1$, with $X_1 \geq 0$. X, in the usual terminology, is an output prospect. $p = (p_1, p_2, \ldots, p_n)$ is an n-dimensional vector of probabilities defined over $\{\theta_i\}$, with p_i standing for the probability estimate of both the landlord and the tenant (see assumption (5) above) that state θ_i will occur (i.e. that an output of X_i will obtain). $p_i \in (0,1) \; \forall i$ and $\sum_1^n p_i = 1$. The mean $\bar{X}$ of the output prospect X is defined by $\bar{X} = \sum_1^n p_i X_i$, and the variance of the distribution of the X_i around $\bar{X}$ $(= \sum_1^n p_i X_i^2 - \bar{X}^2)$ is designated by σ_x^2. In conformity with assumption (4) above, we shall write $\sigma_x^2 = \sigma_x^2(\bar{X})$, $\sigma_x^{2\prime}(\bar{x}) < 0$, i.e. the 'mean-variance' curve will slope downwards rather than upwards as is customary in portfolio-choice theoretic models: this follows from our associating increasing productivity with increasing reliability in irrigation. The fixed rental fell shall be designated by A and it will be assumed that $\exists m \in (1,n): X_m = A$. For future reference, I define the following four vectors:

(1) $X_s^L = (rX_1, rX_2, \ldots, rX_n) \equiv (s_1, s_2, \ldots, s_n)$

which is the output prospect confronted by the landlord under share-cropping;

(2) $X_F^L = (\mathrm{Min}(X_1, A), \mathrm{Min}(X_2, A), \ldots, \mathrm{Min}(X_n, A)) \equiv (F_1, F_2, \ldots, F_n)$

which is the output prospect confronted by the landlord under fixed renting;

(3) $X_S^T = (\rho X_1, \rho X_2, \ldots, \rho X_n) \equiv (R_1, R_2, \ldots, R_n)$

which is the output prospect confronted by the tenant under share-cropping;

(4) $X_F^T = (\mathrm{Max}(0, X_1 - A), \mathrm{Max}(0, X_2 - A), \ldots, \mathrm{Max}(0, X_n - A)) \equiv (G_1, G_2, \ldots, G_n)$.

which is the outp ut prospect confronted by the tenant under fixed-renting.

Next, I define the means and variances of these prospects. (In what follows, E stands for the expected value operator):

(5) $E(X_S^L) \equiv \bar{S} = r\bar{X}$

$Var(X_S^L) = Var(r\bar{X}) = r^2 Var(X) = r^2 \sigma_x^2$

(6) $E(X_F^L) \equiv \bar{F} = \sum_1^m p_i X_i + A \sum_{m+1}^n p_i$

$Var(X_F^L) \equiv \sigma_F^2 = \sum_1^n p_i F_i^2 - \bar{F}^2 = \sum_1^m p_i X_i^2 + A^2 \sum_{m+1}^n p_i - (\sum_1^m p_i X_i + A \sum_{m+1}^n p_i)^2$

(7) $E(X_S^T) \equiv \bar{R} = \rho\bar{X}$

$Var(X_S^T) = Var(\rho X) = \rho^2 Var(X) = \rho^2 \sigma_x^2$

(8) $E(X_F^T) \equiv \bar{G} = \sum_{m+1}^n p_i X_i - A \sum_{m+1}^n p_i$

$Var(X_F^T) \equiv \sigma_G^2 = \sum_1^n p_i G_i^2 - \bar{G}^2 = \sum_{m+1}^n p_i X_i^2 - A^2 \sum_{m+1}^n p_i - (\sum_{m+1}^n p_i X_i - A \sum_{m+1}^n p_i)^2$

Notice that

$$
(9)\quad \begin{cases} E(X_S^T) = E(X) - E(X_S^L) \\ Var(X_S^T) = Var(X) - Var(X_S^L) - 2Cov(X_S^T, X_S^L) \\ E(X_F^T) = E(X) - E(X_F^L) \\ Var(X_F^T) = Var(X) - Var(X_F^L) - 2Cov(X_F^T, X_F^L) \end{cases}
$$

It is useful to note here that given (6), it follows that

$$
(10)\qquad \frac{d\sigma_F^2}{dF} \equiv \frac{d\sigma_F^2}{dA} \Big/ \frac{dF}{dA} = 2(A-F) \geq 0 .
$$

I now define the utility functions of the landlord and the tenant. $U^L(\cdot)$ and $U^T(\cdot)$ are von Neumann -- Morgenstern utility functions of the landlord and the tenant, defined over their respective prospects, say X^L and X^T. $U^L(\cdot)$, $U^T(\cdot)$ are both strictly concave, increasing and indefinitely differentiable functions, and are given by

$$
(11a)\qquad U^L(X^L) = \sum_{1}^{n} P_i u^L(X_i^L) \quad \text{and}
$$

$$
(11b)\qquad U^T(X^T) = \sum_{1}^{n} P_i u^T(X_i^T)
$$

where $u^L(\cdot)$, $u^T(\cdot)$ are utility functions with properties identical to those of $U^L(\cdot)$, $U^T(\cdot)$. (I am assuming in (11a) and (11b) that the form of the individual's utility is not influenced by which state of nature occurs). In what follows, the expected utility functions $U^L(\cdot)$ and $U^T(\cdot)$ will be represented by $Eu^L(\cdot)$ and $Eu^T(\cdot)$ respectively.

Constant relative risk-aversion will be assumed for both landlord and tenant -- viz., that the measure of relative-risk-aversion $\left(= -\frac{u''(x)}{u'(x)}\cdot x\right)$ is invariant with

respect to x, where x is a variable designating output. The logarithmic function satisfies all the properties demanded by $u^L(.)$ and $u^T(.)$, and for reasons of simplicity I shall specialize the utility functions to their logarithmic forms.[3]

Consider now the landlord's expected utility under the two modes of tenancy -- fixed-renting and share-cropping. His expected utility under the two regimes is given by the following expressions, to derive which I have resorted to an approximation of a Taylor's expansion of $Eu^L(X^L_F)$ and $Eu^L(X^L_S)$ around $\bar{F}$ and $\bar{X}$ respectively, and suppressed terms involving powers greater than 2, by assuming a sufficient degree of concentration of the elements of the prospects around their expected values to permit this:[4]

$$(12) \qquad Eu^L(X^L_F) \simeq u^L(\bar{F}) + \frac{1}{2} u^{L''}(\bar{F})\sigma^2_F$$

$$(13) \qquad Eu^L(X^L_S) \simeq u^L(r\bar{X}) + \frac{r^2}{2} u^{L''}(r\bar{X})\sigma^2_x$$

Specialising the utility function $u^L(.)$ to its logarithmic form, (12) and (13) can be written as:

$$(12') \qquad Eu^L(X^L_F) \simeq \log(\bar{F}) - \frac{\sigma^2_F}{2\bar{F}^2}, \text{ and}$$

$$(13') \qquad Eu^L(X^L_S) \simeq \log(r\bar{X}) - \frac{\sigma^2_x}{2\bar{x}^2}$$

It is now clear that the landlord will be indifferent as between the two modes of tenancy if his expected utility from the two modes is the same. The condition for indifference, given (12') and (13') is then given as

(14) $\left(\varphi^L(r,\bar{F})\equiv\right) \quad \log(\bar{F}) - \frac{\sigma_F^2}{2\bar{F}^2} - \left(\log(r\bar{X}) - \frac{\sigma_x^2}{2\bar{X}^2}\right) = 0.$

The curve $\varphi^L(r,\bar{F}) = 0$ is then a locus of combinations of $(r,\bar{F})$ for which the landlord is indifferent as between fixed-renting and share-cropping. Using standard symbols to designate partials and cross-partials, the slope and the rate of change of slope of the $\varphi^L(r,\bar{F}) = 0$ locus can be seen to be given by

(15) $$\left.\frac{dr}{d\bar{F}}\right|_{d\varphi^L=0} = -\frac{\varphi^L_{\bar{F}}}{\varphi^L_r} \quad \text{and}$$

(16) $$\left.\frac{d^2r}{d\bar{F}^2}\right|_{d^2\varphi^L=0} = \frac{(\varphi^L_r)^2\varphi^L_{\bar{F}\bar{F}} + (\varphi^L_{\bar{F}})^2\varphi^L_{rr}}{(\varphi^L_r)^3}$$

Given (10), (12'), (13'), (14), (15) and (16), it may be verified that

(17) $$\left.\frac{dr}{d\bar{F}}\right|_{d\varphi^L=0} = r\varphi^L_{\bar{F}} \quad \text{where}$$

(18) $$\varphi^L_{\bar{F}} = \frac{2}{\bar{F}} + \frac{\sigma_F^2}{\bar{F}^3} - \frac{A}{\bar{F}^2}\ ; \text{ and}$$

(19) $$\left.\frac{d^2r}{d\bar{F}^2}\right|_{d^2\varphi^L=0} = -r\left(\varphi^L_{\bar{F}\bar{F}} + (\varphi^L_{\bar{F}})^2\right) \quad \text{where}$$

(20) $$\varphi^L_{\bar{F}\bar{F}} = \frac{A}{\bar{F}^3} + \frac{2}{\bar{F}^2} + (\varphi^L_{\bar{F}})^2 - \frac{3\varphi_{\bar{F}}}{\bar{F}}$$

Notice now that $\left.\frac{dr}{d\bar{F}}\right|_{d\varphi^L=0} \gtrless 0$ according as $\varphi^L_{\bar{F}} \gtrless 0$, i.e., according as (given (18)) $\frac{2}{\bar{F}} + \frac{\sigma_F^2}{\bar{F}^3} - \frac{A}{\bar{F}^2} \gtrless 0$ which condition, after taking note of the definition of σ_F^2 from (6) can be written as

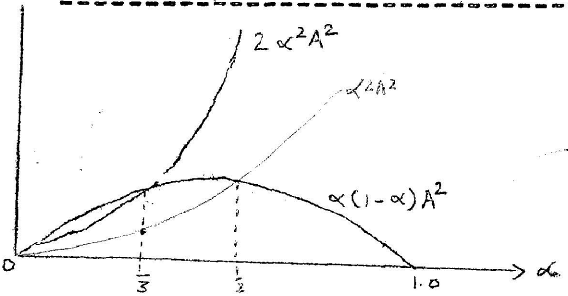
(21) $$A\bar{F} - \bar{F}^2 \lesseqgtr \sum_1^n p_i F_i^2$$

Now we know that $A \gtrsim \bar{F}$. Define $\bar{F} \equiv \alpha A$, $(\alpha \in (0,1))$. Then condition (21) can be rewritten as

$$\text{(21a)} \quad \left[\frac{dr}{d\bar{F}}\bigg|_{d\phi^L=0} \gtrless 0 \right] \leftrightarrow \alpha(1-\alpha)A^2 \gtrless \sum_1^n p_i F_i^2 .$$

It will now be shown that $\frac{dr}{d\bar{F}}\Big|_{d\phi^L} < 0$ (or equivalently, given (21a)), that $\alpha(1-\alpha)A^2 > \sum_1^n p_i F_i^2$ is likely to hold for relatively 'small' values of $\bar{F}$. To see this, consider the expression $\alpha(1-\alpha)A^2$, which is a quadratic function that is double-valued and symmetric about $\alpha = \frac{1}{2}$, and maximised at $\alpha = \frac{1}{2}$. The function is drawn in Figure 1 below

Figure 1 : <u>The $\alpha(1-\alpha)A^2$ and $2\alpha^2 A^2$ functions</u>



Next, note that $\sigma_F^2 \equiv \sum_1^n p_i F_i^2 - \bar{F}^2$, so that, given $\sigma_F^2 \equiv v_F \bar{F}^2$ and $v_F \leq 1$ by assumption, we have

$$\text{(21b)} \quad \bar{F}^2 \equiv \alpha^2A^2 \leq \sum_1^n p_i F_i^2 \leq 2\bar{F}^2 \equiv 2\alpha^2A^2$$

The curves $2\alpha^2A^2$ and α^2A^2 are also plotted in figure 1; the former intersects the curve $\alpha(1-\alpha)A^2$ from below at $\alpha = \frac{1}{3}$, and the latter at $\alpha = \frac{1}{2}$. Note now that $\sum_1^n p_i F_i^2$ cannot anywhere lie above $2\alpha^2A^2$, nor below α^2A^2, as reflected in (21b); furthermore, since $\sum_1^n p_i F_i^2$ is everywhere an exponentially increasing function of α, the curve $\sum_1^n p_i F_i^2$ must intersect the curve $\alpha(1-\alpha)A^2$ uniquely and from below, for some $\alpha \in (\frac{1}{3}, \frac{1}{2}]$. Let the intersection occur for $\alpha = \alpha^*$. Define $\bar{F}^* \equiv \alpha^* A$. Then for $\bar{F} < \bar{F}^*$ (i.e. for relatively 'small' values of $\bar{F}$), $\alpha(1-\alpha)A^2 > \sum_1^n p_i F_i^2$ is satisfied and therefore

$$\frac{dr}{d\bar{F}}\bigg|_{d\phi^L=0} < 0 \text{ will hold. Further,}$$

given (19) and (20), for $\varphi^L_{\bar{F}} < 0$, it is clear that $\varphi^L_{\bar{F}\bar{F}} > 0$ and hence $\frac{d^2r}{d\bar{F}^2}\Big|_{d^2\varphi^L=0} < 0$. Briefly, $\frac{dr}{d\bar{F}}\Big|_{d\varphi^L=0} < 0$ and $\frac{d^2r}{d\bar{F}^2}\Big|_{d^2\varphi^L=0} < 0$ will hold for $\bar{F} < \bar{F}^*$, with $\bar{F}^*$ defined as above.

Next consider the case $\bar{F} > \bar{F}^*$. It is clear straightaway that in this case $\varphi^L_{\bar{F}} > 0$ and therefore that $\frac{dr}{d\bar{F}}\Big|_{d\varphi^L=0} > 0$. Further, given (18), (19), and (20), it can be verified that $\frac{d^2r}{d\bar{F}^2}\Big|_{d^2\varphi^L=0} \gtrless 0$ according as

$$\frac{A}{\bar{F}^3} + \frac{2}{\bar{F}^2} + (\varphi^L_{\bar{F}})^2 - \frac{3\varphi^L_{\bar{F}}}{\bar{F}} + (\varphi^L_{\bar{F}})^2 \gtrless 0$$

Note now that if $\frac{A}{\bar{F}^3} + \frac{2}{\bar{F}^2} + (\varphi^L_{\bar{F}})^2 - \frac{3\varphi^L_{\bar{F}}}{\bar{F}} > 0$, then it will be true, a *fortiori*, that $\frac{A}{\bar{F}^3} + \frac{2}{\bar{F}^2} + (\varphi^L_{\bar{F}})^2 - \frac{3\varphi^L_{\bar{F}}}{\bar{F}} + (\varphi^L_{\bar{F}})^2 > 0$.

Now the requirement of $\frac{A}{\bar{F}^3} + \frac{2}{\bar{F}^2} + (\varphi^L_{\bar{F}})^2 - \frac{3\varphi^L_{\bar{F}}}{\bar{F}} > 0$ can be shown to be equivalent to the requirement:[5]

$$(22) \qquad 2A(v_F - 1) - \frac{\sigma_F^2}{\bar{F}} - \frac{A^2}{\bar{F}} - \left(\frac{\sigma_F^2}{\bar{F}^3}\right)^2 < 0,$$

where v_F is the coefficient of variation of the distribution of the F_i's, with $v_F < 1$ satisfying the requirement of 'small' dispersion. Given $v_F < 1$, it is readily clear that (22) is satisfied, which ensures that $\frac{d^2r}{d\bar{F}^2}\Big|_{d^2\varphi^L=0} < 0$. Briefly, for $\bar{F} > \bar{F}^*$, we have $\frac{dr}{d\bar{F}}\Big|_{d\varphi^L=0} > 0$ and $\frac{d^2r}{d\bar{F}^2}\Big|_{d^2\varphi^L=0} < 0$.

Notice now that for given A, a larger $\bar{F}$ reflects a larger $\bar{X}$ (the productivity per unit size of land) and so a smaller σ_x^2. Holding A fixed, let $\bar{X}^*$ be the value of $\bar{X}$ for which $\bar{F}$ assumes the value $\bar{F}^*$. The results on the sign restrictions on the slope of the landlord's locus of indifference can then be interpreted as follows: for relatively small levels of productivity ($\bar{X} < \bar{X}^*$) and therefore high risk (i.e. high σ_x^2), the $\varphi^L(r, \bar{F}) = 0$ curve is downward-sloping and strictly concave, while for relatively high levels of productivity ($\bar{X} > \bar{X}^*$) and

FIGURES 2(a) & 2(b): LOCI OF INDIFFERENCE BETWEEN SHARE-CROPPING AND FIXED TENANCY FOR LANDLORD AND TENANT AT 'LOW' AND 'HIGH' LEVELS OF PRODUCTIVITY.

FIGURE 2(a):

$\bar{X} < \bar{X}^*$ and downward-sloping loci of indifference: C is the region of mutually compatible equilibria in share-cropping.

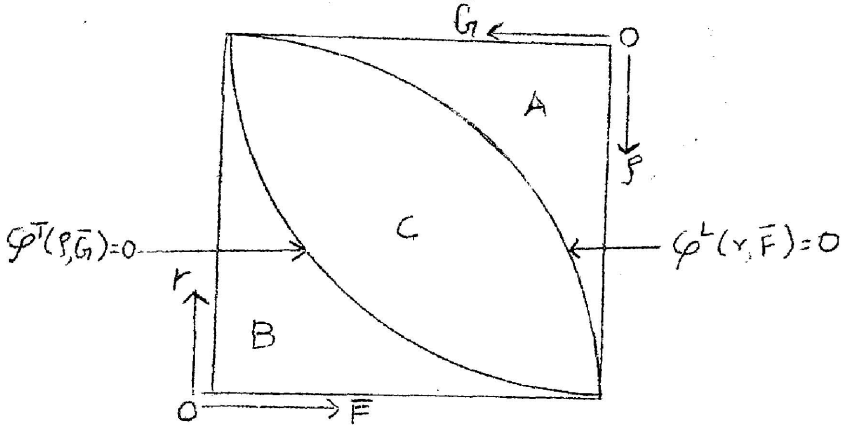
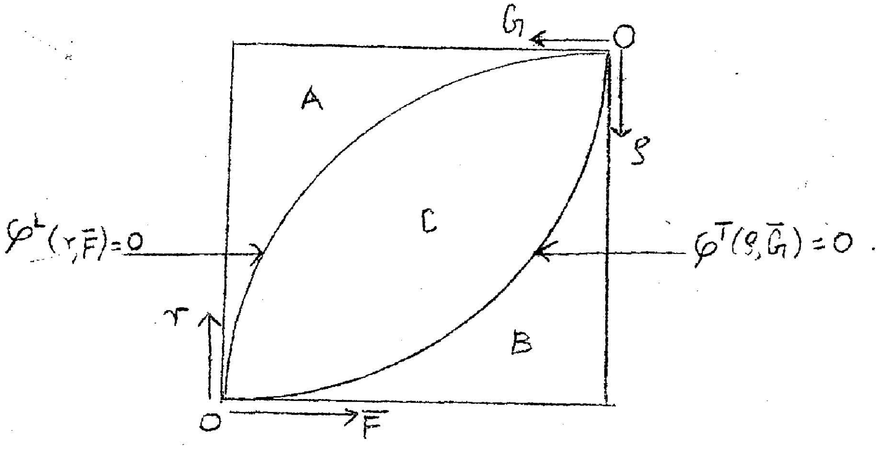


Figure 2(b):

$\bar{X} > \bar{X}^*$ and upward-sloping loci of indifference: C is the zone of mutually compatible equilibria in fixed-renting.



therefore low risk (i.e. low σ_x^2), the $\varphi^L(r,\bar{F})=0$ curve is upward-sloping and stricly concave. The behaviour of the $\varphi^T(\varsigma,\bar{G})=0$ curve (the locus of combinations of $(\varsigma,\bar{G})$ for which the tenant is indifferent as between the two modes of tenancy) may (given (9)) be readily imagined to be identical to the behaviour of the $\varphi^L(r,\bar{F})=0$ curve, by precisely symmetric reasoning.

The two loci of indifference will now together impose restrictions on the set of values in $(r,\bar{F})$ space which ensure mutually compatible equilibria for the landlord and the tenant: this can be most clearly seen if one employs a diagrammatic apparatus of the Edgeworth-box type depicted in figures 2(a) and 2(b), which correspond to the restrictions on the slopes of the indifference loci for 'low' and 'high' values of $\bar{X}$.

In figures 2(a) and 2(b), we measure $\bar{F}$ along the horizontal axis and r along the vertical axis with the lower left-hand vertex of the box as origin: $\bar{F}$ and r are measured from west to east and from south to north respectively. Similarly, with the upper right-hand vertex of the box as origin, we measure $\bar{G}$ from east to west along the horizontal axis and ς from noth to south along the vertical axis. The $\varphi^L(r,\bar{F})=0$ and $\varphi^T(\varsigma,\bar{G})=0$ curves are then plotted in each of the figures.

Figures 2(a) and 2(b) are drawn for $\bar{X}<\bar{X}^*$ and for $\bar{X}>\bar{X}^*$ respectively. In figure 2(a), the landlord will prefer fixed renting in the region above his locus of indifference, i.e. in the region A, while the tenant will prefer fixed-renting in the region above his locus of indifference, i.e. in the region B. Quite clearly, no mutually compatible equilbria in fixed- renting are possible; on the other hand, since both tenant and landlord will favour share-cropping in the region below

their respective loci of indifference, this region-- marked C in the figure -- is the only one which allows for the possibility of mutually compatible equilibria, and that, too, in share-cropping. In figure 2(b), *per contra*, the landlord will prefer fixed-renting in the region *below* his locus of indifference, and like-wise the tenant will prefer fixed-renting in the region below *his* locus of indifference : there are no mutually compatible equilibria possible in share-cropping, and zone C represents the only region of mutually compatible equilibria, and that, too, in fixed-renting.

Briefly, for $\bar{X} < \bar{X}^*$ (i.e. for 'low' levels of productivity and 'high' risk), one finds that the tenancy arrangement likely to prevail is share-cropping; and for $\bar{X} > \bar{X}^*$ (i.e. for 'high' levels of productivity and low risk), the tenancy arrangement likely to prevail is fixed-renting. This tendency for the mode of tenancy to progress from the share-cropping arrangement to the fixed-renting arrangement as agriculture stabilises was, it will be recalled, precisely the empirical possibility suggested by the data in Table 1 of this note, that I set out to find a theoretical rationale for in this Section.

III *On an Equilibrium Rental Share of one-half: a Brief Extension*

In section II of this note it was assumed (see assumption (1)) that the landlord (tenant) had already resolved his problem of choice between leasing out (in) land and not leasing out (in) land in favour of the former. In this brief section, I explore the resolution of this choice-problem in the context of share-cropping in slightly greater detail.

Consider first the landlord's problem. He can exercise one of three options: (i) to do nothing, in which case his expected utility will be zero; (ii) to cultivate his land himself, in which case, if λ represents his share of output net of cultivation costs, his expected utility will be given by $\log(\lambda\bar{X}) - \frac{\sigma_x^2}{2\bar{X}^2}$; and (iii) to lease out his land on a share-cropping basis, in which case his expected utility (as we have already seen) will be given by $\log(r\bar{X}) - \frac{\sigma_x^2}{2\bar{X}^2}$. Now clearly, a necessary condition for self-cultivation to be worthwhile to the landlord is that λ must satisfy

(23) $$\log(\lambda\bar{X}) - \frac{\sigma_x^2}{2\bar{X}^2} \geqq 0$$

For the landlord to be indifferent between self-cultivation and leasing out his land on a share-cropping basis, r must satisfy

(24) $$r = \lambda$$

Given (23) and (24), the landlord will find it just worthwhile to go in for share-renting if

(25) $$\log(r\bar{x}) - \frac{\sigma_x^2}{2\bar{x}^2} = 0,$$

from which one can obtain the landlord's reservation rental-rate as

(26) $$r^s = (\exp.(\tfrac{v}{2}))/\bar{X} \quad , \quad (v \equiv \frac{\sigma_x^2}{\bar{X}^2})$$

Notice now that variations in the level of productivity of land will, given (26), ensure the existence of an upward-sloping market-supply curve for land leased out under share-cropping.

Consider next the problem from the tenant's point of view. In the absence of a competitively perfect labour-market in which the option to hire out his labour at a (positive) wage-rate is available, the tenant will find it worth his while to lease in land on a share-cropping basis only if his expected utility under share-cropping is non-negative, i.e. only when ρ satisfies

(27) $\log(\rho\bar{x}) - \frac{\sigma_x^2}{2\bar{x}^2} \geqq 0$.

Recalling that $\rho \equiv 1-r$, and given (27), the tenant's reservation rental-rate can be derived as

(28) $r^d = 1 - (\exp.(\frac{V}{2}))/\bar{x}$

From (26) and (28), we obtain

(29) $r^s = 1 - r^d$

Notice again that variations in the level of productivity of land will, given (28), ensure the existence of a downward-sloping market-demand curve for land leased in under share-cropping. Given an upward-slopping supply-curve and a downward-slopping demand curve, if the two curves intersect, then the resulting equilibrium will be unique and determinate. In the event of such an equilibrium, the demand and supply prices will coincide with the equilibrium price r^e at equilibrium:

(30) $r^e = r^s = r^d$

From (29) and (30) one immediately obtains

(31) $r^e = \frac{1}{2}$,

which is the second of the two empirically observed phenomena for which a theoretical rationale was wanting.

Concluding Observations

In this note I have attempted to employ certain fairly standard tools belonging to the theory of decision-making under risk in order to explore a possible logical foundation for a couple of empirically observed phenomena relating to the institution of tenancy. In the course of doing this exercise, I have had to resort to a number of restrictive, simplifying assumptions: at best, a set of sufficient conditions have been established

which ensure that the analytical predictions of the theoretical framework I have employed are broadly compatible with certain impressions based on observation. It is possible that a more generalized model with more 'realistic' assumptions than the one presented in this note could be constructed to explain the phenomena under study. Indeed, it is possible that the explanation for these phenomena is to be located entirely elsewhere than within the confines of the framework of the model outlined here. In any event, it needs to be emphasized that the exercise in this note addresses itself to treating 'rationality' only as ' ... an assumption to be explored and not a hypothesis that is tested.'[6]

Notes

1/ Data on tenancy in the World Agricultural Census (1970/71) are of a highly disaggregated nature. For each panchayat union of each taluk of each district, we have information on the number and area, for each size-class, of three different types of operational holdings: (a) holdings which are wholly owned and self-operated; (b) holdings which are wholly rented; and (c) holdings which are partly owned and partly rented. Under category (b) of tenure we have a further disaggregation in terms of the number and area of holdings under fixed-money tenancy, under fixed-produce tenancy, under share-of-produce tenancy and under a fourth, undefined category simply called 'others'. It should be noted that the data in the World Agricultural Census are unsatisfactory for a number of reasons which have been documented elsewhere (see V.K. Ramachandran [2]), and which we need not go into here. Nevertheless, the data presented in Table 1 of this note do seem to be broadly indicative.

2/ See, for example, Rudra [3]

3/ For our purposes, it is not essential that $u^{L}(.)$ and

$u^T(.)$ be identical; it is sufficient, for example, that one be a positive affine transform of the other.

4/ In particular, we shall assume that the coefficients of variation v_F and v_S of the two prospects are less than unity.

5/ To see that $\frac{A}{\bar{F}^3} + \frac{2}{\bar{F}^2} + (\varphi^L_{\bar{F}})^2 - \frac{3\varphi^L_{\bar{F}}}{\bar{F}} > 0 \leftrightarrow 2A(v_F - 1) - \frac{\sigma_F^2}{\bar{F}} - \frac{(\sigma_F^2)^2}{\bar{F}^3} - \frac{A^2}{\bar{F}} < 0,$

Consider the following:

$$\frac{A}{\bar{F}^3} + \frac{2}{\bar{F}^2} + (\varphi^L_{\bar{F}})^2 - \frac{3\varphi^L_{\bar{F}}}{\bar{F}} > 0$$

$$\leftrightarrow \frac{3\varphi^L_{\bar{F}}}{\bar{F}} - \frac{A}{\bar{F}^3} - \frac{2}{\bar{F}^2} - (\varphi^L_{\bar{F}})^2 < 0$$

$$\leftrightarrow 3\bar{F}^2\varphi^L_{\bar{F}} - A - 2\bar{F} - \bar{F}^3(\varphi^L_{\bar{F}})^2 < 0 \text{ (after multiplying through by } \bar{F}^3\text{)}$$

$$\leftrightarrow 3\bar{F}^2\left(\frac{2}{\bar{F}} + \frac{\sigma_F^2}{\bar{F}^3} - \frac{A}{\bar{F}^2}\right) - A - 2\bar{F} - \bar{F}^3\left(\frac{2}{\bar{F}} + \frac{\sigma_F^2}{\bar{F}^3} - \frac{A}{\bar{F}^2}\right)^2 < 0 \text{ (after substituting out } \varphi^L_{\bar{F}} \text{ from (18))}$$

$$\leftrightarrow 2A\frac{\sigma_F^2}{\bar{F}^2} - 2A - \frac{\sigma_F^2}{\bar{F}} - \frac{A^2}{\bar{F}} - \left(\frac{\sigma_F^2}{\bar{F}^3}\right)^2 < 0 \text{ (upon simplification)}$$

$$\leftrightarrow 2A(v_F - 1) - \frac{\sigma_F^2}{\bar{F}} - \frac{A^2}{\bar{F}} - \frac{(\sigma_F^2)^2}{\bar{F}^2} < 0$$

6/ Quoted from Sen [4]

References

[1] Cheung, S.N.S. 'Transaction Costs, Risk Aversion and the Choice of Contractual Arrangements', J.Law Econ., April 1969, 12, 23-42.

[2] Ramachandran, V.K. 'A Note on the Sources of Official Data on Land Holdings in Tamil Nadu'. Data Series No.1, Madras Institute of Development Studies, 1980.

[3] Rudra, A 'Share Cropping in East Bengal', Economic and Political Weekly, Vol.10, No.27, September 27, 1975.

[4] Sen, A.K. 'peasants and Dualism with or without Surplus Labour', Journal of Political Economy, Volume LXXIV, October 1966, p425-450.